



**COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING DEPARTMENT**

EENG 577

**ADVANCED ELECTRICAL MACHINE DYNAMICS
FOR SMART-GRID SYSTEMS**

**M7-1 Switched Reluctance Motor Drive Principles of Operation
& State Space Models**

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Switched Reluctance Motor Drive Systems

- In this module, Switched Reluctance Motor, SRM, drive system principles of operation are presented.
- In addition, methods for inverter modeling, developed torque calculations, state space representation, and an overview of a case study, are discussed.

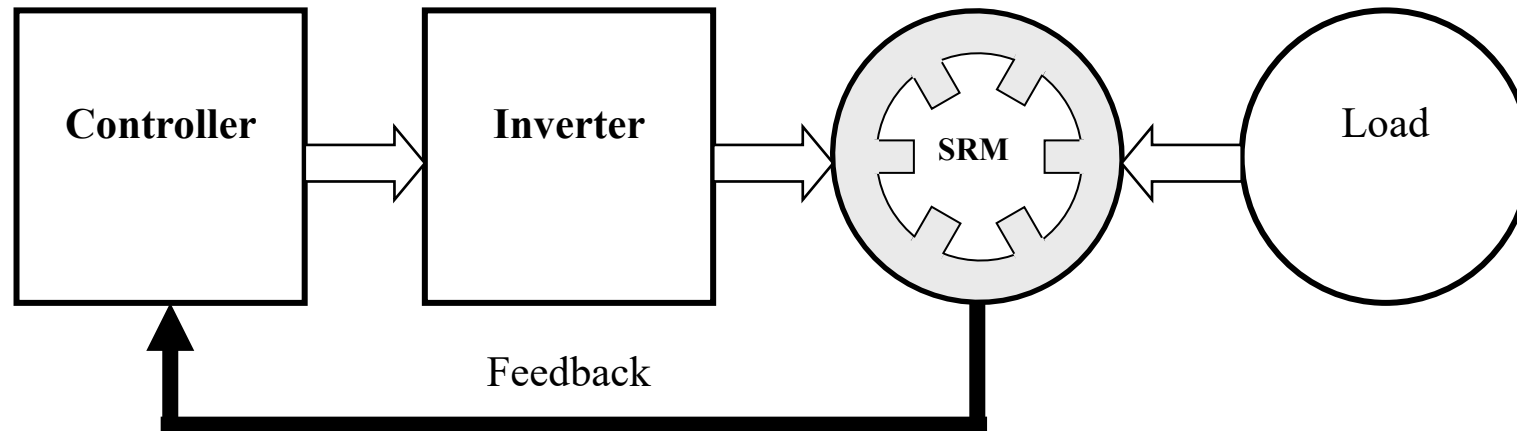


Figure 1. Block Diagram of SRM and Drive System

1. SRM Drive System Description

- A Switched Reluctance Motor, SRM, drive system is usually composed of three major components as well as the load. Those components or parts are the SRM itself, the inverter, and the controller.
- The controller is used to determine the switching sequence of the inverter, which in turn supplies the motor with the needed power.
- The inverter power electronics switching is based on an algorithm employed in the controller as well as the SRM rotor position, which is fed back to the controller from the motor position sensors.
- In this study, a prototype was used which involves a 6/4, 0.15 hp, 5000 r/min SRM drive system and a hysteresis break which was used to load the motor. Check references [1-8] for full details. A schematic of an SRM drive system described is shown in the block diagram of Figure 1.

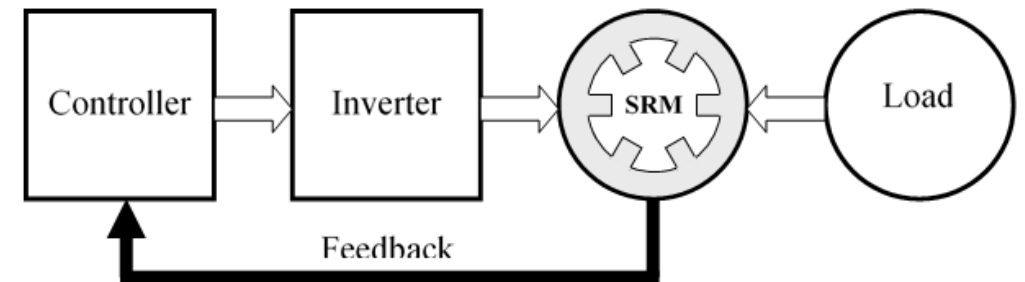


Figure 1. Block Diagram of SRM and Drive System

- SRMs were developed in the late 1800's but have seen little use in practical applications. They are among the simplest of all rotating machinery.
- These motors have no form of excitation on the rotor, which eliminates the need for brushes or slip rings, and this type of motor is quite rugged and durable with little need for servicing or upkeep. Despite these apparent advantages over other types of motors, SRMs have not been widely used until the 1980's due to the difficulties involved in their control.
- However, with the advances in control technology and the availability of power electronic components, switched reluctance motors are being developed for a wide variety of applications as an alternative to other types of motors where a high degree of reliability is required.
- Because of this, much of the earlier literature concerning switched reluctance motors was centered on the development of low-cost drive systems and effective methods for characterizing and controlling the devices.

- SRMs are categorized according to the number of poles (teeth) on the stator and the number of the poles on rotor. The machine involved in this work has 6 stator poles, and 4 rotor poles. Accordingly, the machine is called a 6/4 SRM.
- In addition, the motor has three phases designated as ***a***, ***b***, and ***c***. In the SRM considered in this work, each phase is constructed by connecting in series the windings of two opposite poles on the stator.
- In this manner an additive flux density is produced throughout the rotor steel. Figure 2 shows a cross-section of the 6/4 SRM under consideration.

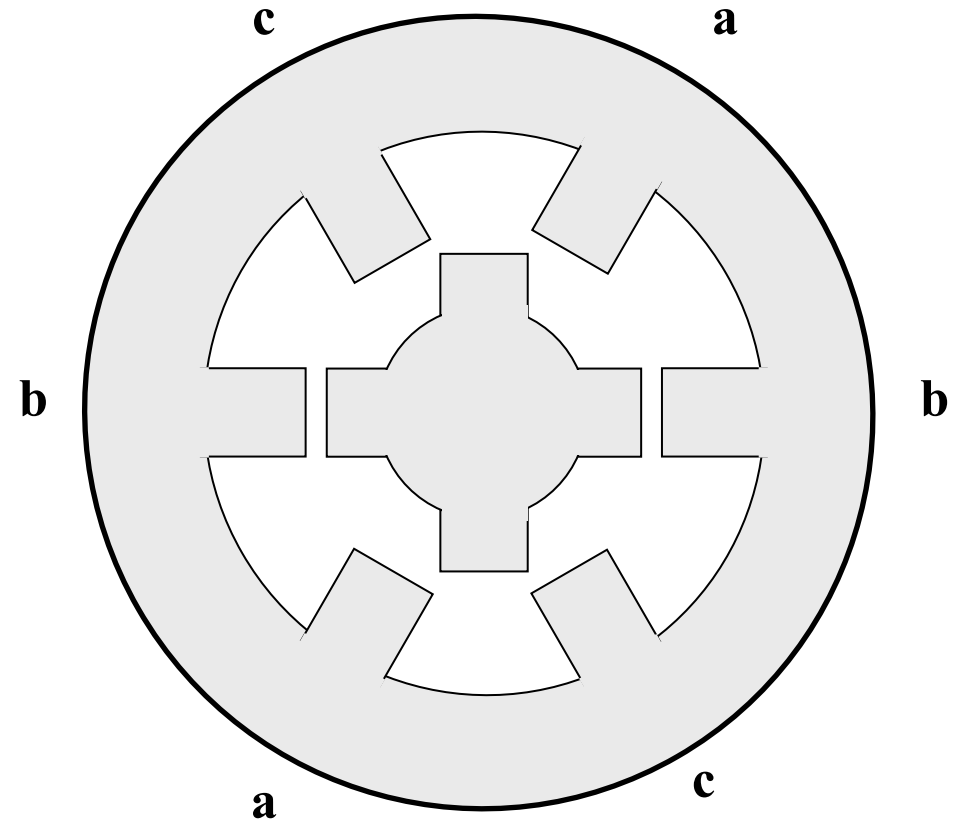


Figure 2. A 6/4 SRM Cross-Section

2. The Principals of SRM Operation

- The torque development in SRMs is based on the reluctance reduction concept. The torque is developed when the rotor moves from one position to another in order to reduce reluctance.
- The rotor movement is always from a position with maximum reluctance to a position where the reluctance is minimal when a particular phase is excited. The operation of the SRM can be explained better by considering Figure 3.

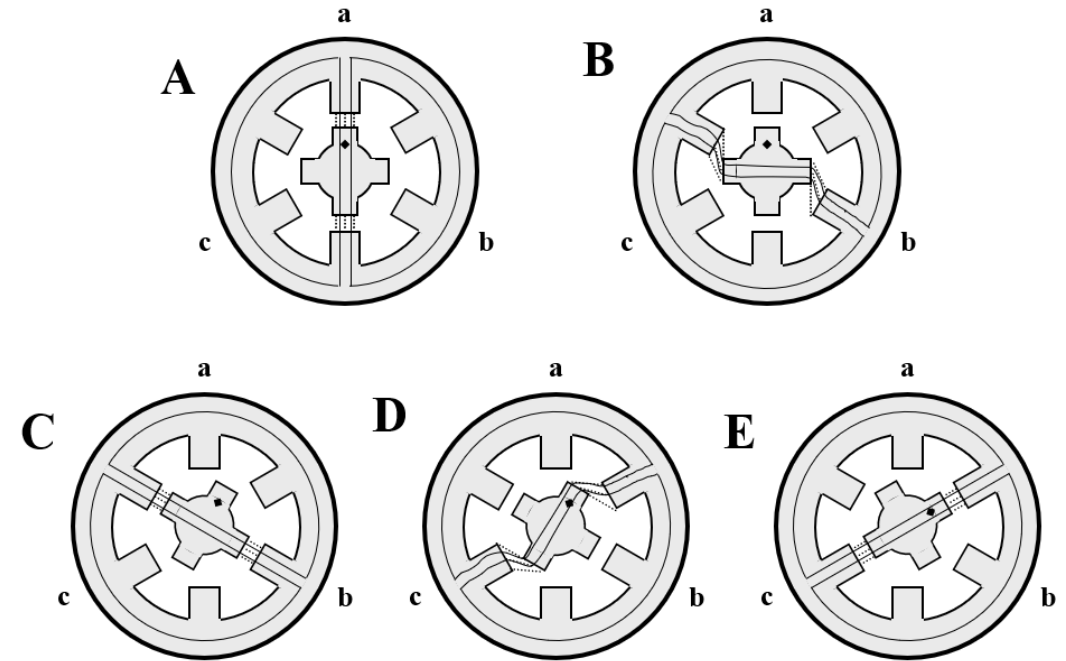
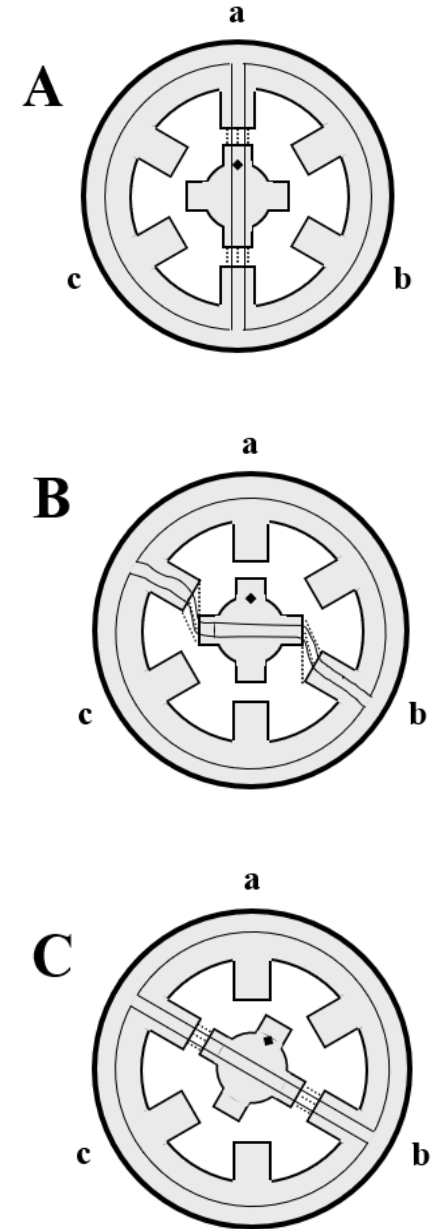


Figure 3. 6/4 SRM Cross-Sections for Various Rotor Positions to Show the Torque Development
(Airgap is exaggerated to show the flux resulting from the excited phase)

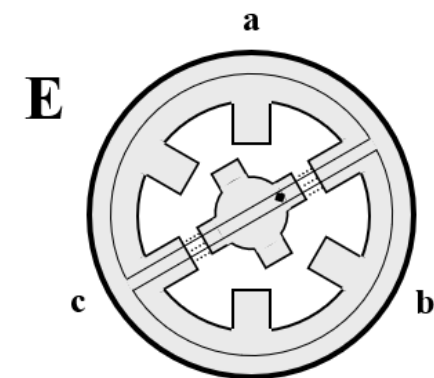
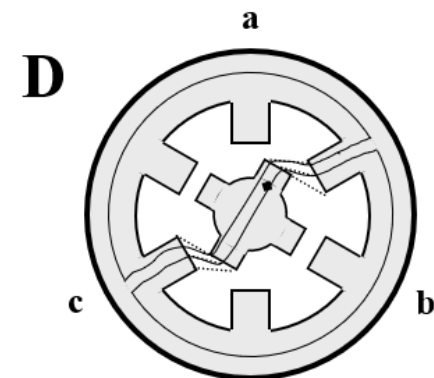
- Figure 3-A shows the position of the rotor corresponding to the case when phase *a* is powered or excited. In this case, phase *a* corresponding windings on the stator poles are energized. As a result, and in order to reduce the reluctance of the corresponding magnetic circuit, the rotor poles closest to phase *a* poles are lined up with it to achieve a minimum reluctance position.
- If phase *a* is turned off (after achieving the minimum reluctance position) and phase *b* is turned on, a new magnetic circuit is created Figure 3-B. An inspection of this magnetic circuit reveals that the rotor is not in a stable position. The rotor shaft is forced to move from its position to another position where the rotor poles are lined up with the poles of phase *b* (again where the reluctance is minimal), Figure 3-C.



- Similarly, if phase ***b*** is turned off and phase ***c*** is turned on, the rotor moves to the position of corresponding minimal reluctance as shown in Figures 3-D and 3-E. As can be noted, the movement of the rotor shown in Figure 3 is clockwise.
- The sequence of phase excitation used was ***a***, ***b***, then ***c***. The rotation of the rotor can be counterclockwise by changing the sequence to ***a***, ***c***, then ***b***.
- The rotation of the rotor is governed by the following expression:

$$\theta = \frac{2\pi}{mNr} = \frac{360^\circ}{mNr} \quad (1)$$

where m is the number of phases and Nr is the number of rotor poles. For the motor concerned in this work, $m = 3$ and $Nr = 4$, hence $\theta = 30^\circ$. As such, one phase switching will result in a rotation of 30° mechanical.



3. SRM Developed Torque Calculation

- As was stated before, in this machine, the torque is produced by forcing the rotor to move by energizing the windings of the diametrically opposite pairs of stator poles one at a time (switching the phases).
- Considering an idealized SRM where both steel saturation and the mutual inductances are neglected, and the self-inductance only is taken into consideration. The developed torque can be expressed in terms of the rate of change of co-energy with respect to a small perturbation of the rotor position as follows:

$$T = \frac{\partial W'(i_a, i_b, i_c, \theta)}{\partial \theta} \quad (2)$$

where i_a , i_b , and i_c are the currents in the phases a, b, and c respectively and θ is the rotor position.

- The co-energy W' can be expressed in terms of the phase self inductance as follows:

$$W' = \frac{1}{2} [L_{aa}(\theta)i_a^2 + L_{bb}(\theta)i_b^2 + L_{cc}(\theta)i_c^2] \quad (3)$$

where $L_{aa} = L_{bb} = L_{cc}$ are the self-inductances of phase a, b, and c, respectively.

- If one treats the currents in (3) as constants in the partial differentiation with respect to θ , the expression of the developed torque in Equation (2) can be written as:

$$T = \frac{1}{2} \frac{dL}{d\theta} \sum_{j=a,b,c} i_j^2 \quad (4)$$

- Equation (4) shows clearly that the developed torque is a function of the square of the phase current values and the derivative of the self-inductance with respect to the rotor position θ .

- If one holds the armature currents invariant, the average torque value will come out zero. This phenomenon is due to the nature of the armature self-inductances and corresponding derivatives.
- Therefore, the production of positive torque requires phase current variation with respect to the rotor position and the rate of change of the self-inductance with respect to rotor position to be positive.
- This is illustrated in Figure 4 where the current waveforms are plotted along with the inductance profiles, and the corresponding derivatives.
- Note that the inductance profiles are not idealized as it can be seen from their non-smooth nature. As a result, the corresponding derivatives are also not smooth, and one expects a non-smooth torque profile for this class of machines.

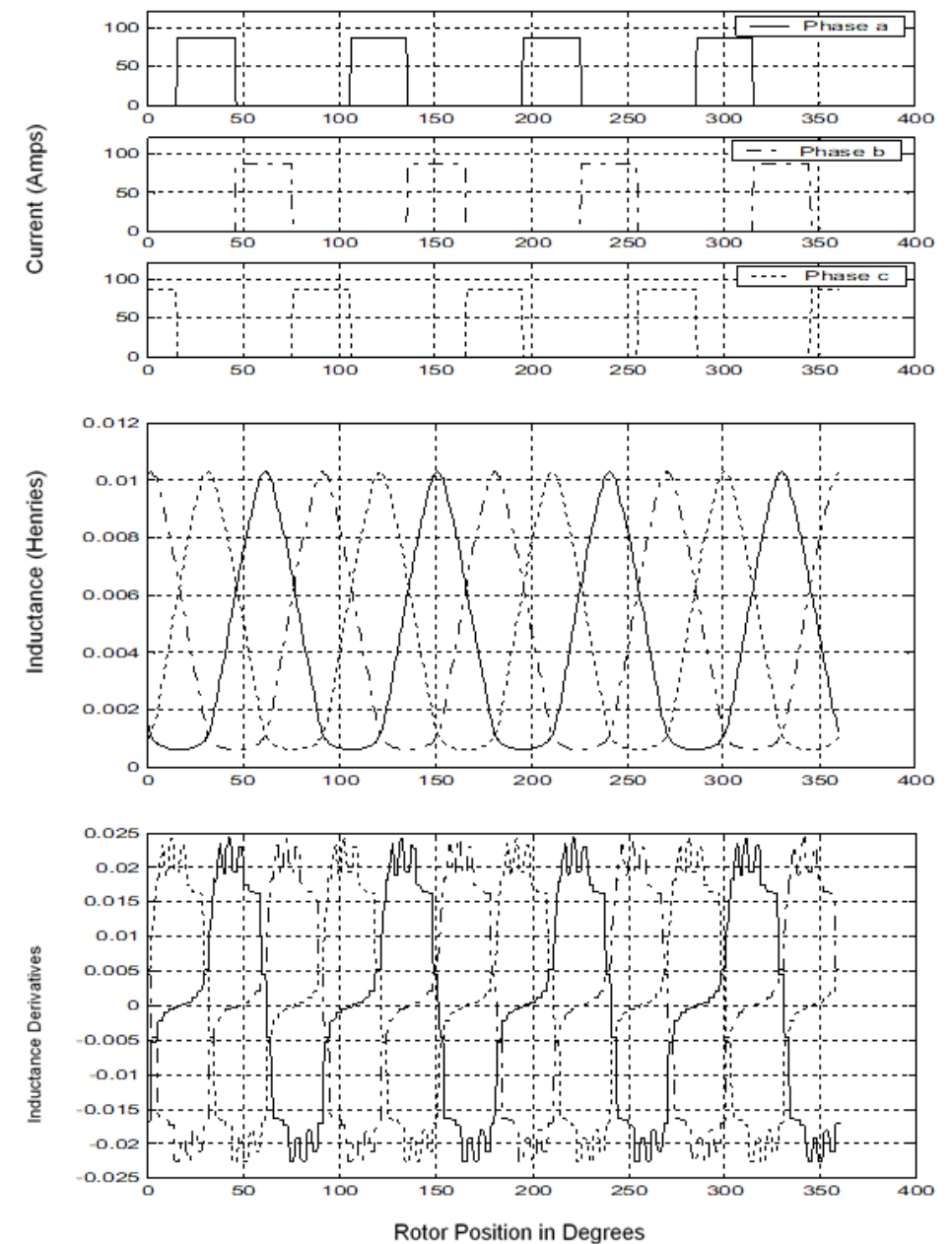


Figure 4: Current Waveforms, Inductance Profiles, and Corresponding Derivative

4. State Space Representation of the SRM

- The accurate performance prediction of SRM drive systems, as well as all complex systems, is not possible without the availability of a mathematical representation.
- Starting with a virtual circuit realization of the motor and its drive, one can arrive at a mathematical representation or the state space model that represents the drive system.
- In general, the differential equations used to represent the transient and dynamic performance of electric machines are derived from the interaction among the phase windings.
- The machine under consideration (SRM) has no windings around the rotor poles. The only windings in the motor are the ones around the stator poles. Therefore, the lumped inductance and resistance of the stator windings are the only parameters of the representing circuit.
- In addition to the motor, the drive system consists of the controller and the inverter, which excites the motor stator phases.

- Three inductances in series with resistances, as illustrated in Figure 5, can represent the circuit realization of the stator three windings.
- Figure 5 shows the three windings in an abc frame of reference. The d-q axis is shown for space reference.

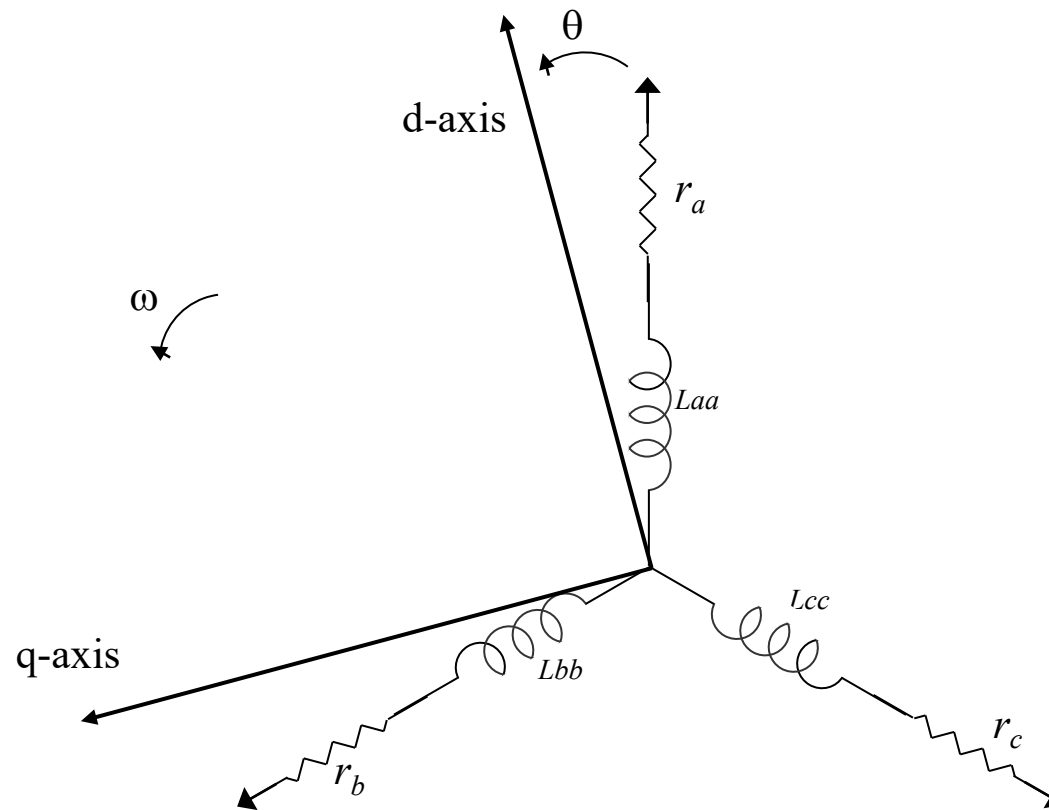


Figure 5: Schematic Representation of Coils

- Applying Kirchhoff's voltage law, the voltage over any phase a, b, or c can be described as follows:

$$v_k = r_k i_k + d\lambda_k/dt \quad (5)$$

where k denotes phases a, b, and c, v_k is the voltage over each phase, λ_k is the flux linkage, r_k is the phase winding resistance, and i_k is the phase current, Figure 5.

- The flux linkage λ_k can be represented in terms of the machine phase currents and inductances as follows:

$$\lambda_j = \sum l_{jk}(i_a, i_b, i_c, \theta) i_k \quad j, k = a, b, c \quad (6)$$

where l_{jk} represents the mutual inductance when $j \neq k$ and the self-inductance when $j = k$.

- As can be appreciated from Equation (6), the SRM self and the mutual inductances are functions of phase current and rotor position.

- From Equations (5) and (6), phase j voltage can be re-written as follows:

$$v_j = r_j i_j + (\sum_k [\frac{\partial \lambda_j}{\partial i_k} \frac{di_k}{dt}]) + [\frac{\partial \lambda_j}{\partial \theta} \frac{d\theta}{dt}] \quad (7)$$

where k spans phases a, b, and c. Also, r_j is the phase resistance and λ_j is the flux linkage for phase j , and θ is the rotor position relative to a fixed axis of reference.

- As shown in references [1]-[3], the rate of change of inductance with respect to current value can be ignored, meaning the derivative of the inductance with respect to the current is zero ($\partial l_{jk}/\partial i_k=0$). As such, one can consider the following relationship:

$$\frac{\partial \lambda_j}{\partial i_k} = l_{jk} \quad (8)$$

In addition,

$$\frac{d\theta}{dt} = \omega_m \quad (9)$$

where ω_m is the mechanical speed in radians per second.

- Substituting Equations (8) and (9) into (7) results in the following for a phase j voltage:

$$v_j = r_j i_j + (\sum_k [l_{jk} \frac{di_k}{dt}]) + [\frac{\partial \lambda_j}{\partial \theta} \omega_m] \quad (10)$$

where k spans the stator windings, a, b, and c.

- Using Equation (10) for each of the SRM stator phases would result in the SRM state space equations in compact matrix form:

$$V = RI + L \frac{dI}{dt} + \omega_m \frac{dL}{d\theta} I \quad (11)$$

- In Equation (11), the first term to the right represents the Ohmic voltage component, the second represents the transformer voltage component, and the last term represents the reluctance, or torque production, voltage component.

In Equation (11) L is a 3x3 inductance matrix of the form:

$$L = \begin{bmatrix} l_{aa} & l_{ab} & l_{ac} \\ l_{ba} & l_{bb} & l_{bc} \\ l_{ca} & l_{cb} & l_{cc} \end{bmatrix} \quad (12)$$

V is the stator phase voltage vector; $V = \begin{bmatrix} v_a \\ v_b \\ v_d \end{bmatrix}$ (13)

R is the stator phase resistance matrix; $R = \begin{bmatrix} r_a & 0 & 0 \\ 0 & r_b & 0 \\ 0 & 0 & r_c \end{bmatrix}$ (14)

I is the stator phase currents vector; $I = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$ (15)

- In addition to (11), the mathematical representation of the drive system requires the mechanical equation. The relation between the torque load and angular velocity of the machine can be described by the following mechanical differential equation:

$$\frac{d\omega_m}{dt} = \frac{1}{J}(T_{em} - B\omega_m - T_L) \quad (15)$$

where J is the moment of inertia of the rotor, T_{em} the electromechanical developed torque, B the viscous coefficient of friction, T_L the load torque applied to the machine shaft, and ω_m is the machine mechanical rotational speed expressed in radians per second.

- As noted in references [1]-[3], SRM stator mutual inductances can be ignored as their effect on the SRM drive system simulated performance characteristics are negligible.

5. Inverter Modeling

- In this section, the details of developing a lumped parameter circuit representation of the SRM and its inverter are given.
- The inverter is a main component of a drive system. The role of the inverter is to supply power to the motor. Accordingly, it is essential that the state space model of SRM include a representation of this inverter.
- The inverter employed in this work is of the H-bridge type. As the rotor has no windings, one can represent the SRM by the stator phases *a*, *b*, and *c* and their corresponding voltage drops V_a , V_b , and V_c , respectively.

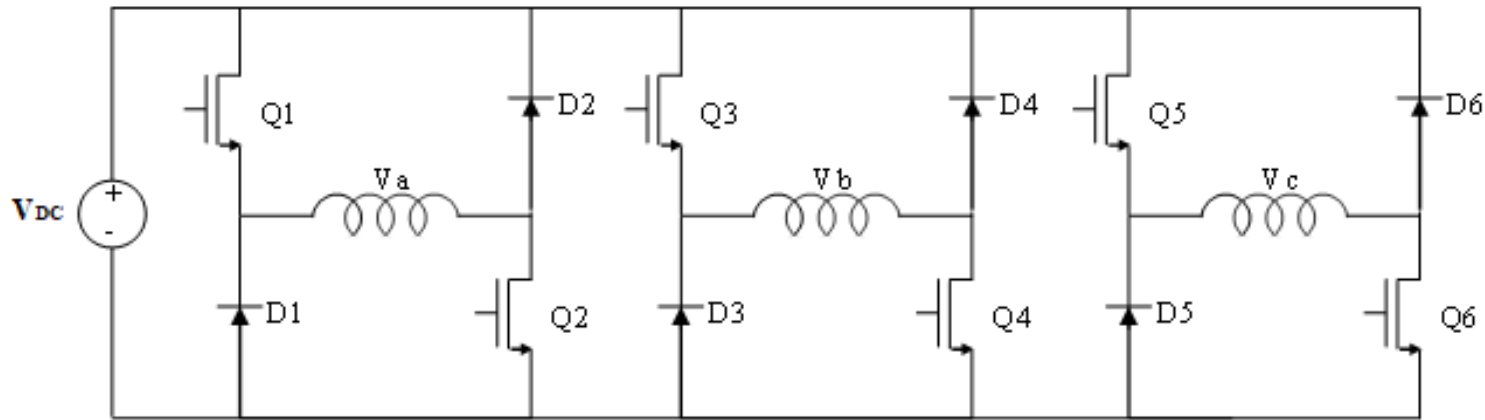


Figure 6. SRM H-bridge Drive Circuit

- The realization of this inverter can be done by freewheeling diodes and MOSFET transistors as shown in Figure 6.
- Each of the three phases of the SRM considered are excited at a specific firing angle (certain angles of the rotor position with respect to the fixed reference of the phases on the stator.)
- Since the motor is a three-phase motor and each phase requires two stator poles, the phases are geometrically 60° apart from each other. In addition, the rotor has four poles, and they are 90° apart from each other, Figure 7.

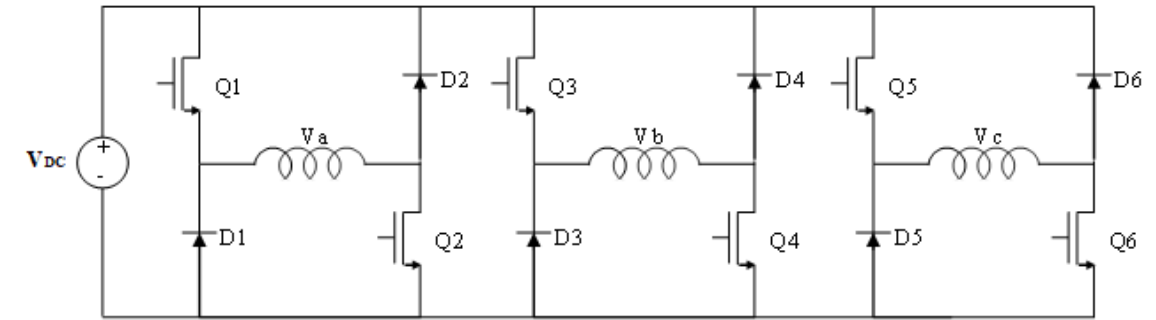


Figure 6. SRM H-bridge Drive Circuit

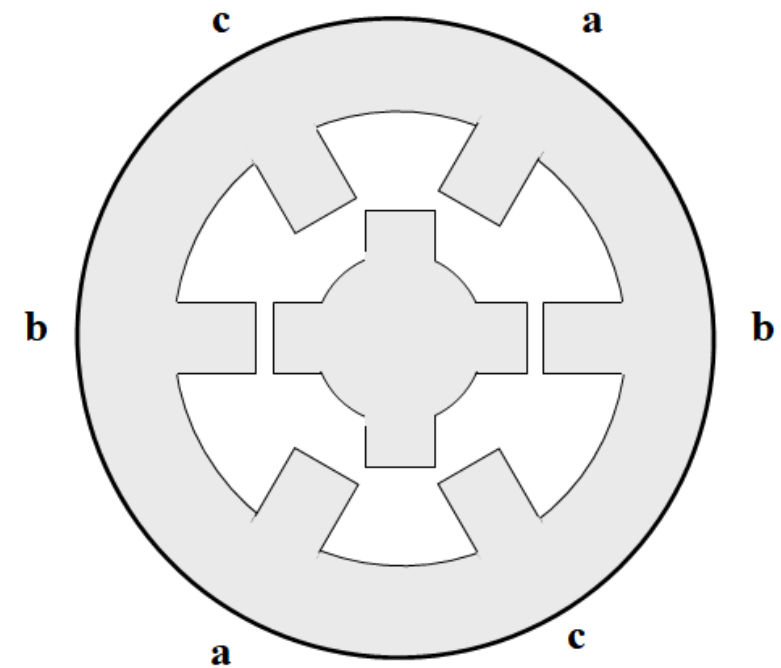


Figure 7. A 6/4 SRM Cross-Section

- To better understand the firing sequence, assume that the reference of the poles on the rotor is initially at an angle θ_f (firing angle) from phase *a*, Figure (7 a). When phase *a* is fired the pole will move to line up with the stator pole of that phase.
- After the pole moves 30° from the initial angle position, another pole will be at θ_f from the other phase (phase *b*). This happens because phase *a* on the stator is 120° apart from phase *b* and the rotor poles are 90° degrees away from each other.

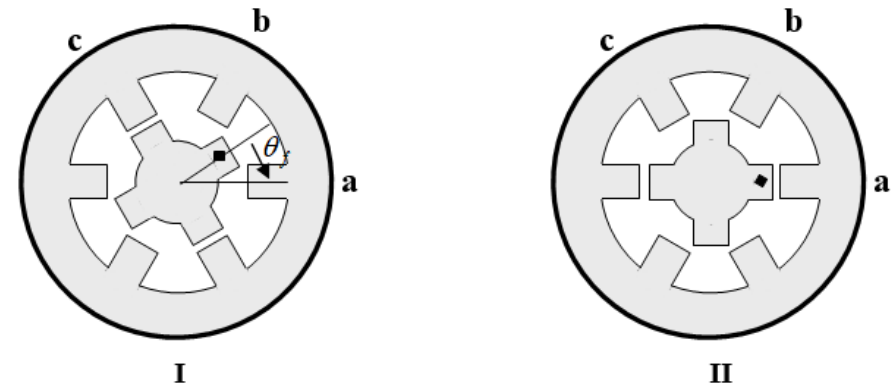


Figure 7 a: Phase *a* Before and After Excitation

- Next, phase ***b*** is to be turned on and phase ***a*** to be turned off, Figure (7 b).
- The same applies to the other phases when the rotor moves 30° degrees again, Figure (7 c).

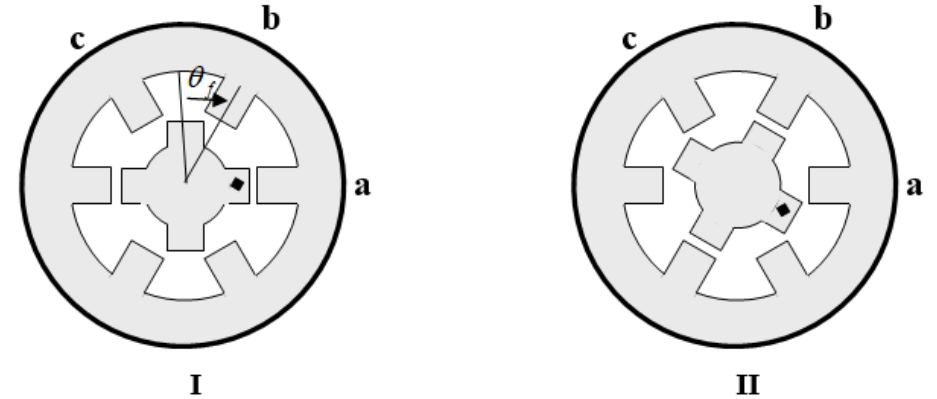


Figure 7 b: Phase ***b*** Before and After Excitation

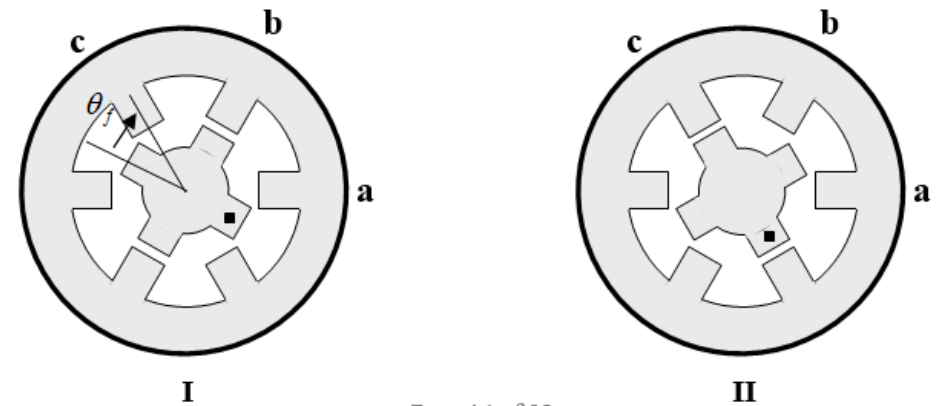


Figure 7 c: Phase ***c*** Before and After Excitation

- The switching sequence used is shown in Figure 8.
- As it can be seen in Figure 8 only one phase is energized at a time.
- The structure of SRM motors dictates energizing one phase at a time to prevent the other phases from producing negative torque.
- Furthermore, once a phase is turned on, the other two should be pulled to a zero current. Accordingly, the H-bridge drive circuit realization depicted in Figure 6 is not only responsible for switching the phases, but also it was designed to drain the current in the unenergized phases.

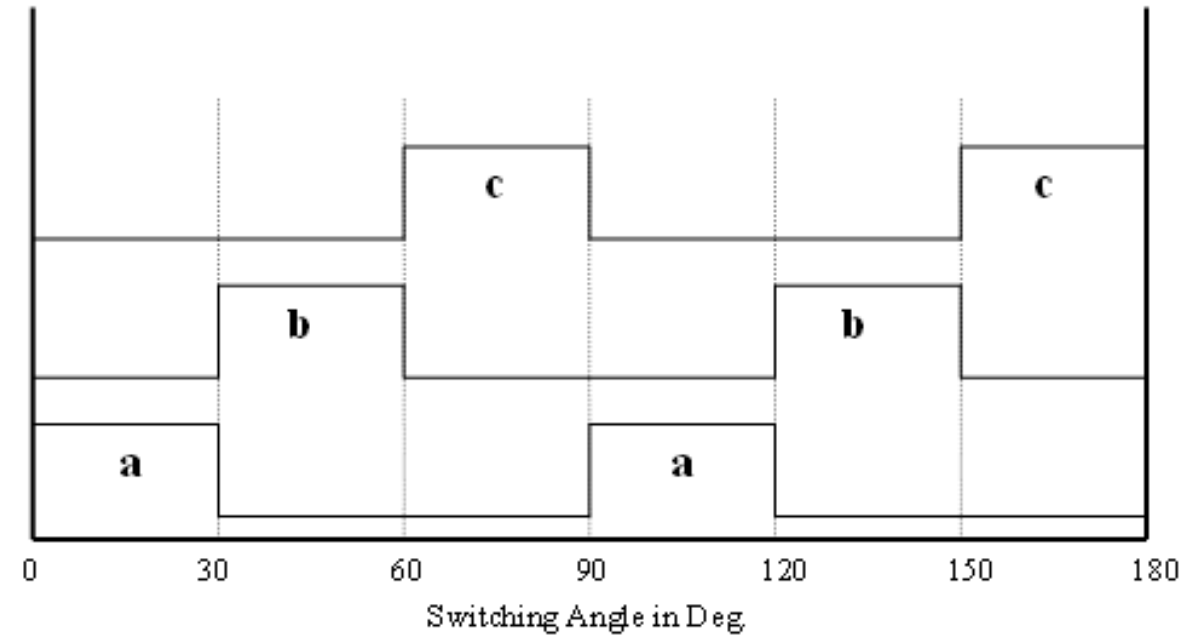


Figure 8. SRM Switching Sequence

- To properly explain the operation of this inverter, we consider the period “0-30°” of Figure 8 corresponding to having phase *a* exited. It is noted that before the switching of phase *a* commences, phase *c* was exited to initialize the sequence and to have the rotor at the position indicated in Figure 7 a.
- When phase *c* was on, the transistors Q5 and Q6 of Figure 6 were on. The current had a path from the $+V_{DC}$ of the source to the ground passing through the coil corresponding to phase *c* in the positive direction according to the +/- assigned to it. At that time, the diodes D5 and D6 of Figure 6 were off, acting like an open circuit.
- Once the rotor moves 30° phase *a* should be energized. In addition, phase *c* should have zero volts. By turning Q1 and Q2 on, the current now will have path through the coil corresponding to phase *a* and consequently, the phase *a* is on. Now the diodes D1 and D2 are off, acting like an open circuit. At the time phase *a* is turned on, the transistors Q5 and Q6 are turned off (Phase *c* is turned off.)

- In this mode phase c will have a negative voltage. The current will circulate through the diode D6, the power source, and the diode D5. This circulation will take place until the remnant current in coil c is dragged to zero. The inverter conduction modes are shown in Figure 9. It is to be noted that the devices and corresponding path of current for each mode of operation in Figure 9 is denoted by a solid line.

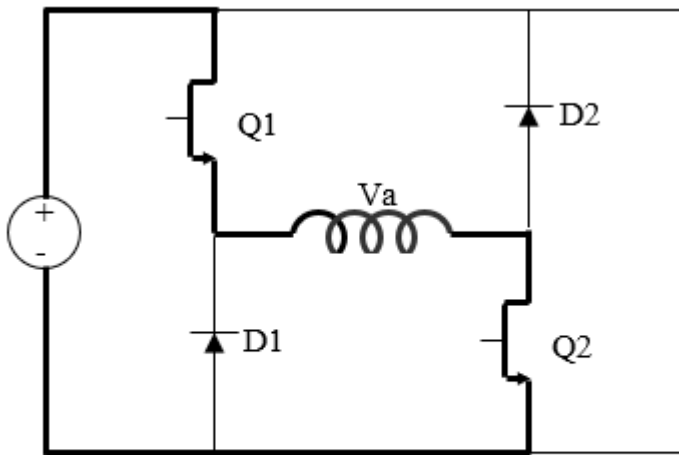


Figure 9-a: Conduction mode of Excitation "on" as shown for phase a

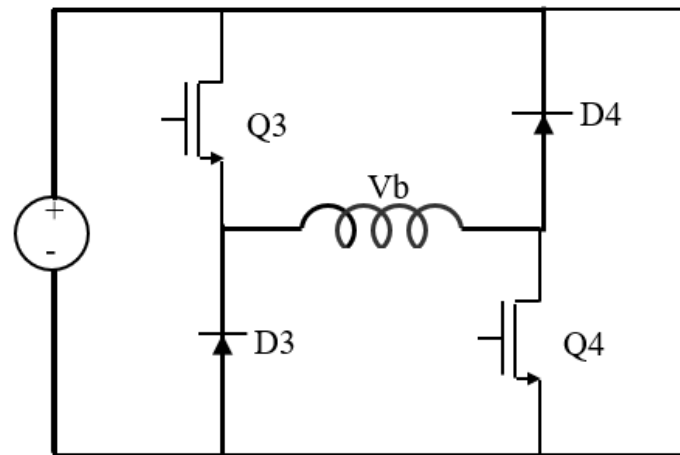


Figure 9-b: Conduction in Reverse Mode "off" as shown for phase b

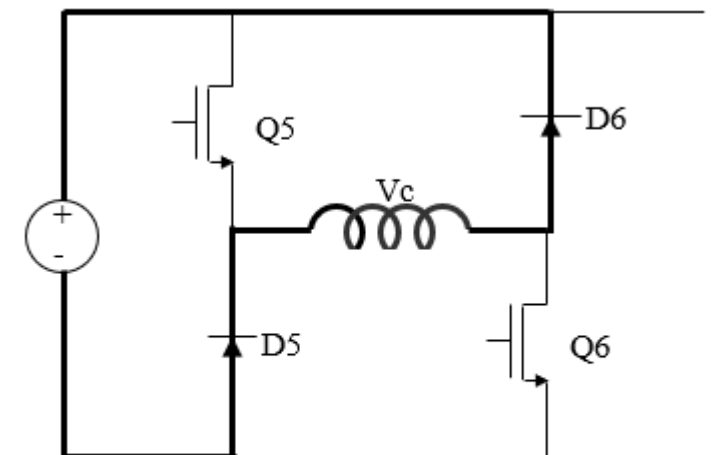


Figure 9-c: Conduction in Reverse Mode "off" as shown for phase c

- For illustration, Figure 10 shows the drive circuit when phase *a* is on and phase *b* and *c* are off.

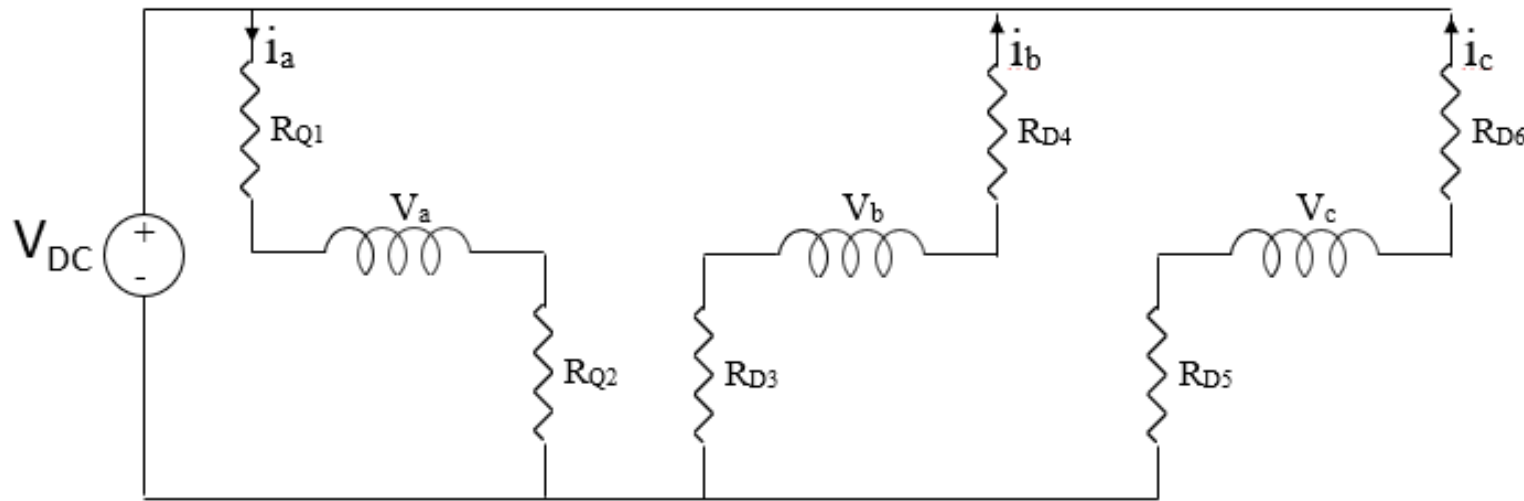


Figure 10. An Inverter Equivalent Circuit

Finally, in an inverter simulation, the states “*on*” and “*off*” for transistors and diodes can be represented using *low and high resistance values*, respectively. As a result, the loop equation can be written as:

- Phase **a**:

$$V_{DC} = R_{Q1}i_a + v_a + R_{Q2}i_a \quad (16)$$

- Phase **b**:

$$V_{DC} = - (R_{D3}i_b + v_b + R_{D4}i_b) \quad (17)$$

- Phase **c**:

$$V_{DC} = - (R_{D5}i_c + v_c + R_{D6}i_c) \quad (18)$$

where R_{Qi} is the equivalent resistance of the transistor Qi and R_{Di} is the equivalent resistance of the diode Di in the on state. Also, v_a, v_b , and v_c are the values of phase voltage for phases **a**, **b**, and **c**, respectively, as given in Equation 13.

6. SIMULINK Model

- The inverter model used in this work was developed using MATLAB/SIMULINK.
- It is comprised of several functional blocks and block connections. Each block has its input(s), output(s) and characteristic function. The idea of those blocks is to create elements that work separately as units and together as a module to represent a certain task.
- The module that simulates the phase switching (one phase) is shown in Figure 11. (Three modules of phase switching were needed.)

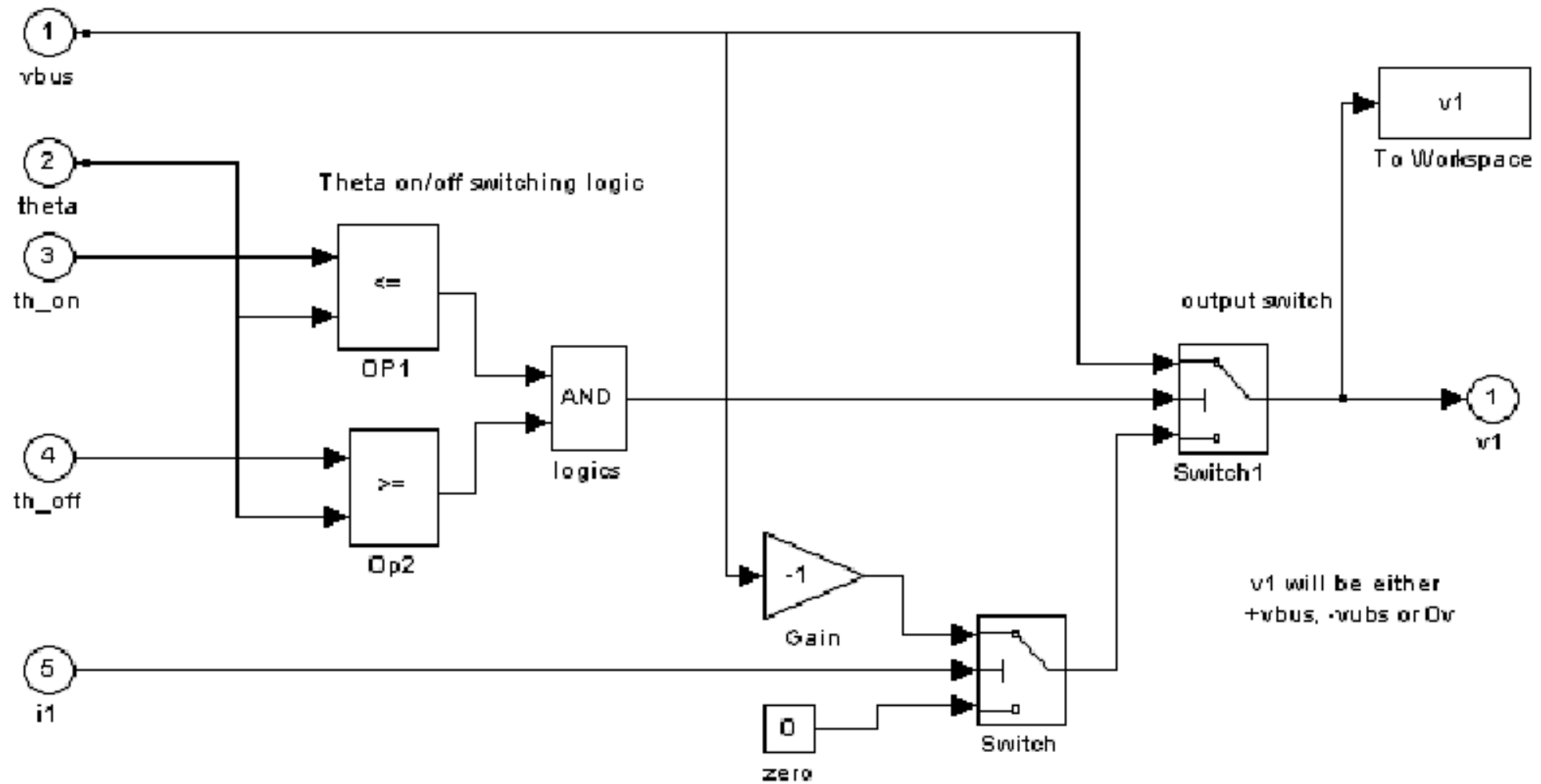
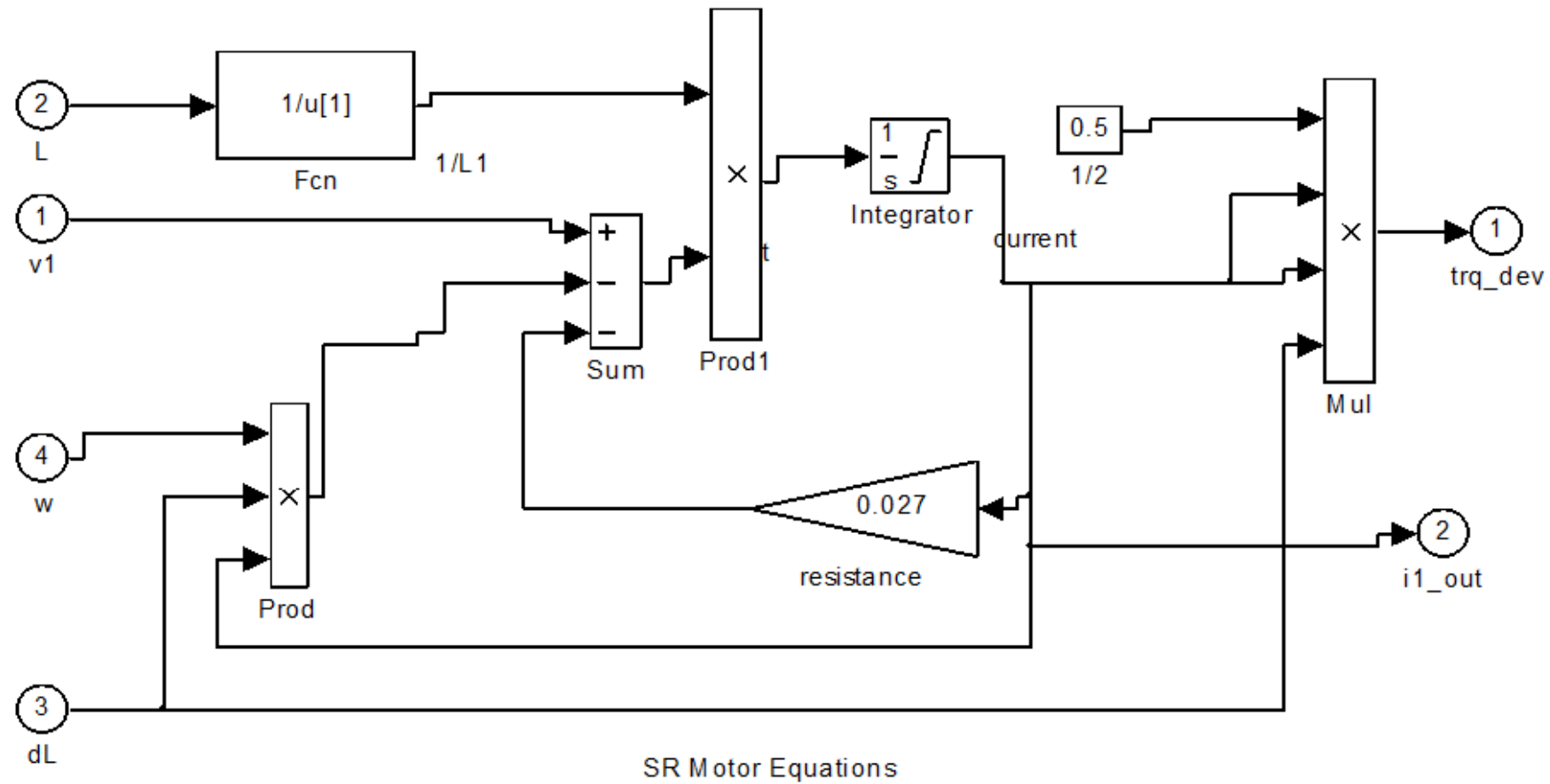


Figure 11: SIMULINK Module to Implement Phase Switching

- In addition, an implementation to represent the motor was provided and integrated with the inverter model to obtain a complete state space model.
- The SIMULINK model of the motor was simply a translation of SS model of Equation (11) and the developed torque of Equation (4) into the SIMULINK elements for each phase.
- In other words, the motor model was composed of three modules each one corresponded to one phase. Figure 12 shows one phase implementation of Equation (11) and the developed torque.
- Each of those modules is used for the calculation of the current and then the developed torque for the corresponding phase. The inverter modules are connected to the motor modules for the voltage values fed-in.



$$v_1 = R \cdot i_1 + w \cdot (dL/d\theta) + L \cdot (di/dt)$$

or $di/dt = (v_1 - R \cdot i_1 - w \cdot (dL/d\theta)) / L$

$$trq_dev = 1/2 \cdot i_1^2 \cdot dL/d\theta$$

Figure 12: SIMULINK Module to Implement a Phase Voltage and Developed Torque

A Case Study on SRM Drive System

- In this case study on a Switched Reluctance Motor, SRM, drive systems, the results of using state space models to predict a motor-drive system dynamic performance characteristics are presented. The results of the study are published in references [1] – [3]:
- In reference [1], the dynamic performance characteristics under normal operating conditions were presented. Using this approach, the state space model parameters were determined from series of nonlinear magnetic field solutions, thus accounting for magnetic material nonlinearities and space harmonics due to the motor geometry.
- The method was applied to a 6/4, 0.15 hp, 5000 r/min SRM and resulted in the machine inductances, which compared favorably to measured values. Using these parameters in the state space model, the dynamic performance characteristics of the motor drive system were predicted and verified by comparison to experimental data. In addition, the effects of mutual coupling between motor phases on the analysis results were evaluated.

- In reference [2], the results of using an iterative approach to predict the dynamic performance characteristics of a 6/4, 0.15 hp, 5000 r/min SRM drive system during different fault conditions were presented.
- This included a transistor failure in the converter and an internal fault due to partial armature phase short.
- The analysis resulted in the performance characteristics of the motor drive system which were verified by comparison to test data.
- In addition, the effects of mutual coupling between motor phases on the analysis results were evaluated.
- In reference [3], the effects of compression on the magnetic properties of a steel alloy used in electromagnetic devices was investigated and the results of a case study of an SRM were presented.
- It was demonstrated that compression affects the winding inductances profiles as well as the performance characteristics of the motor. The simulated performance results were also verified by comparison to test data.

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